# Method General Approach Performance Profiling (Method-GAPP)

MSc. G.H. Hendriksen

gerwin.hendriksen@gmail.com

http://method-gapp.com

http://blog.gerwinhendriksen.com

March 2013, the Netherlands

# **Contents**

C	Contents	2
1	1 Preface	4
2	2 Introduction	5
3	3 What is "Method-GAPP"	6
4	4 "Method-R"	8
5	5 Response time and coherency	9
6	6 "Method-GAPP" Data Collection	10
	6.1 Primary components data	10
	6.2 Secondary components data	11
	6.3 Basic principles	11
	6.4 Data sources to be used	11
	6.5 Values Examination	11
7	7 "Method-GAPP" Data Synchronization	12
•	7.1 Workload profiles identification	
8	8 "Method-GAPP" Data Mining – Explain	13
	8.1 Some simple tests with Swingbench	13
	8.1.1 First Swingbench Test (on Windows 7)	13
	8.2 Using the Factorial Analysis on Primary components	16
	8.2.1 Second Swingbench Test (on Mac)	
	8.3 Important rules regarding component choosing	19
	8.4 Using the Factorial Analysis on Secondary components	19
	8.4.1 SQL Statement response time	20
	8.4.2 Events	22
9	9 "Method-GAPP" Data Modeling	23
1(	10 "Method-GAPP" Data Mining – Model	25
1:	11 "Method-GAPP" Data Mining – Predict	31
12	12 "Method-GAPP" Data Interpretation	32
	12.1 Customer Case: Banking Application	
	12.2 Customer Case: Time and Labor Application	
13	13 Conclusions	39
1	14 References	40

#### Revision

Version	Date	What
1.0	03-2011	Initial Version
1.1	03-2013	Updated

#### 1 Preface

After having worked for more than 15 years with Oracle products and specialized in performance, I developed through time a new performance method I baptized "Method-GAPP". "GAPP" is an abbreviation of "General Approach Performance Profiling". The method makes smart use of underlying queuing models and data mining to find bottlenecks in complex architectures, for specific end-user processes within an enterprise. Another refer can be the "gap" of not having enough tracing data that can be jumped. In base the method uses data, which is most of the time already in place, and the method itself is technology independent.

At the Hotsos Symposium 2008 in Dallas (march 2008), I presented "Method-GAPP" the first time. At that time I had already worked over 6 years on the idea of making it possible to do predictions on end-user performance, based on background metrics. The last couple of years the method has been further enhanced by creating linear models based on queuing theory. These latest steps made predictions on technical architectural changes much easier to be done.

When reading this white paper I hope you get a good impression on how the method works. If you have still question on the data gathering, the calculations, the modeling and or other questions, you can always checkout the information on the "Method-GAPP" website, or you can ask me directly.

Although there are in the market some methods and tools, which might have some similarities with "Method-GAPP", I can ensure you that the method was completely developed by my own effort, and has not been build on other methods / tools. After all I hope you will enjoy the white paper and be impressed by its potential.

Special thanks I like to give to Cary Millsap and Dr. Neil Gunther for their inspiration and support.

Regards,

Gerwin Hendriksen March 2013

#### 2 Introduction

The complexity of IT-infrastructures has increased over the years. The number of applications inside an enterprise has increased and this has lead to more consolidations, more shared components, more virtualization, etc. Also the complexity of applications has introduced more tiers, like a web sever, an application server, an authentication server, a database server, a SAN, etc. It gets harder and harder to be able to find performance bottlenecks for end-user processes. In the old days we could trace a client process and see what the session was doing on a database server and in this way follow where wait time (queuing time) has went in the underlying infrastructure. Nowadays most applications work via a web based client what makes tracing of the process much harder, although of course not impossible.

If we are in the end able to trace such a client process (the "chain"), we still are confronted with external influences we can't explain easy. For example a storage or network, which is used by several applications (shared) within the enterprise. If we trace we can find out that sometimes the I/O is faster and sometimes the I/O is slower, but we are not able to say where this influence is coming from. It would be nice if there were a way to get more insight into this. In this case "Method-GAPP" can be of great help, because it is able to find also influences of applications and components outside your "chain".

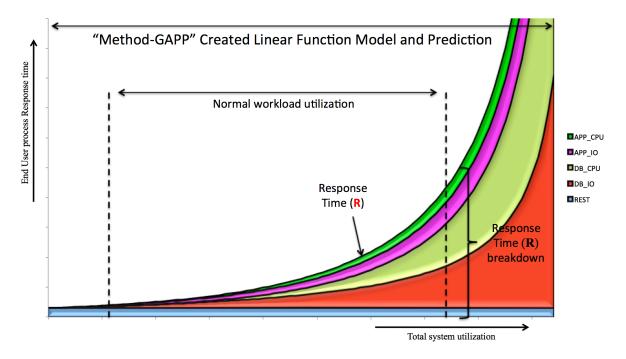
Sometimes we are in an enterprise and questions are raised if we should do a certain investment in hardware, to solve a certain performance problem. In these cases we have to be careful and make sure we really know what the performance bottleneck actually is for the end-user process, to prevent doing a disinvestment. For these questions the method can give you indicating prediction models of future scenario's before the actual investment has happened.

In most cases we are not allowed to trace actual in the application, and / or not allowed to put hooks in the application to checkout where our response time was spent within the architecture. This is most of the time the case in applications from third party vendors. Also in these cases the method can be of great help because it can use simply retrieved data to do its predictions and see what is from influence and in which way. Data that can be used:

- Operating system data from all architecture tiers (e.g. OEM, sar, perfmon, nmon files)
- Database data (e.g. AWR, ASH, ADDM, stats pack)
- SAN metadata
- Web server metadata
- Response time data from the business processes (e.g. Mercury Loadrunner data, Moniforce data, Oracle RUEI, APM tools like AppDynamics, etc.)
- Instances metadata from BPEL Process manager (e.g. SOA Suite, etc.)
- Message data from ESB, etc.
- Business performance indicator data (e.g. BAM tools, etc.)
- Etc.

#### 3 What is "Method-GAPP"

"Method-GAPP" is an abbreviation of Method "General Approach Performance Profiling". The method makes smart use of underlying queuing models and data mining to find bottlenecks in complex architectures, for specific end-user processes within an enterprise. The method can be used to performance profile business (end-user) processes on high level, but depending on the components you put in, even on detailed level. In a simplified way "Method-GAPP" can do the following using only the so called primary components:



Graph 1: In a simplified way Method-GAPP can be used to create a linear function model of the response time (R) for an end-user process based on the normal workload utilization of the system. When having the model it is very simple to break down the end-user response time (R) and see what the impact of each component is. Further more the model can predict the end-user response time (R) of situations the total system (total technical infrastructure) never has encountered before.

The method uses data mining to determine which components are the most involved in the end user response time or throughput of a business process. Based on the found components, models are created using queuing theory, which makes it possible to breakdown the end user response time or throughput in the involved components and predict future scenarios.

To use "Method-GAPP" you have to know that the method uses five important steps, the five "D's":

- Data Collection
- Data Synchronization
- Data Modeling
- Data Mining
- **D**ata Interpretation

These five steps can be written as the following actions to perform the method for a specific to be predicted end user response time or throughput:

- 1. Data Collection
- 2. Values Examination
- 3. Data Synchronization
- 4. Workload profile identification
- 5. Primary component data:
  - a. Factorial analysis (attribute importance, data mining explain)
  - b. Data Modeling by usage of data mining
  - c. Data mining linear function creation
  - d. Interpretation
- 6. Secondary component data:
  - a. Factorial analysis (attribute importance, data mining explain)
  - b. (Optional) Data Modeling by usage of data mining
  - c. (Optional) Data mining linear function creation
  - d. Interpretation

The mentioned steps and actions will be further described in the white paper. To understand better how "Method-GAPP" works, we will start how "Method-R" from Cary Millsap is describing influences on response time. This way of looking at the response time is an important viewpoint for "Method-GAPP".

#### 4 "Method-R"

"Method-R" (*Millsap 2003*) describes how the "chain" of tiers in a technical infrastructure influences end-user processes. Diagram 1 shows different end-user processes response times (R) been influenced by different tiers of a technical infrastructure. Important to realize is the fact that the response time (R) of each individual tier is determined by the wait / queuing time (Q) and the service time (S), so R=Q+S. Further that the end-user process response time (R) is basically the sum of all the individual tier response times, so:  $R=R_{AS}+R_{NET}+R_{DB}+R_{SAN}+R_{STOR}$ . For example the purple process on the application tier shows some queuing time (Q) and service time (S) only on the application server, but for the red process all tiers are used. So tunings effort on the database server will not have any effect on the purple process, but will have influence on the red process, etc.

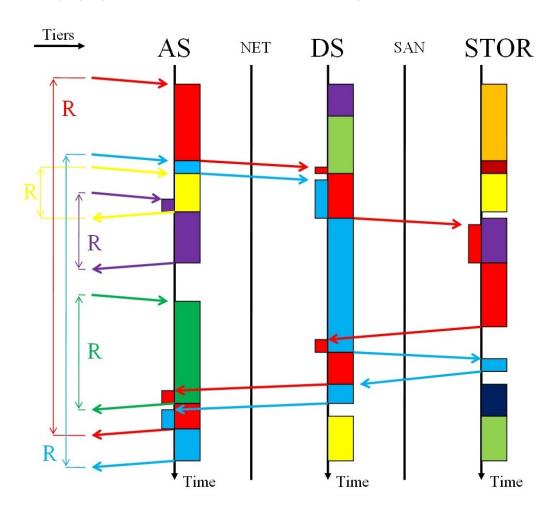


Diagram 1: Sequence diagram to make clear how Cary Millsap's Method-R (*Millsap 2003*) is describing how different processes (different colors), are using the architecture in different ways. In the picture are five tiers mentioned: Application Server (AS), Network (NET), Database Server (DS), Storage Area Network (SAN) and Storage (STOR). For the network (NET) and the storage area network (SAN) is only a line drawn, to make the diagram simpler. The blocks on the left side of the line is queuing time (Q) and on the right side of the line service time (S), for each tier.

## 5 Response time and coherency

To go a little further for understanding what "Method-GAPP" is doing we have to realize that the response time (R) is not only influenced by the queuing time (Q), but also by the stretching service time (S), due to coherency. Coherency is the effect that although you are serviced, you're not doing real work, because you're waiting for coordination with another process. The effect is that service time (S) gets stretched, e.g. trashing of the CPU cache, trashing of SAN cache, etc. With queuing time (Q) you actually are in the queue waiting to be serviced. See Diagram 2 how the actual impact of the response time (R) variance is build up.

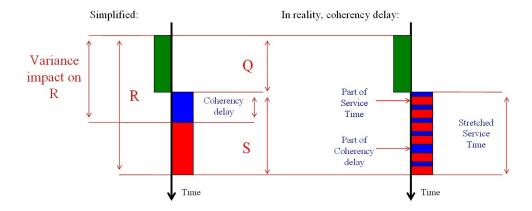


Diagram 2: Diagram to make clear how the variance of response time (R) of a specific part of the infrastructure (e.g. CPU resource), is influenced not only by queuing time (Q), but also the stretching of service time due to coherency. In real the coherency is not just one block in the service time, but small repeating delays.

So basically we can say that for each end-user process the difference between fast and slow response time (R) execution can be described as a diagram like shown in diagram 3:

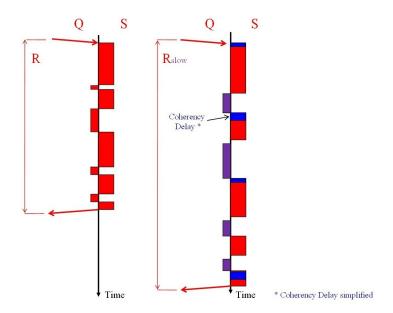


Diagram 3: Diagram to make clear how changes of queuing time (Q) and coherency can make total response (R) time longer (Rslow).

#### 6 "Method-GAPP" Data Collection

Based on the knowledge we discussed we can start thinking of what data should be collected for an analyses. If we are collecting data we need data of each tier in the technical architecture of the "chain" (see Method-R), but also data from tiers not part of the "chain" can be used as part of the analyses to checkout if there is influence or not on the response time (R) or throughput. In Illustration 1 is shown how the response time graph (red) can possibly be described by several metrics (blue), it is only not clear which metric or combination of metrics is describing the response time graph the best.

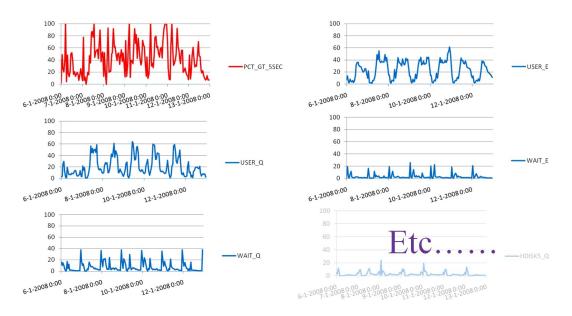


Illustration 1: Illustration to show how the response time (red color) should be possible to be explained by metrics of the technical infrastructure.

When start to collect data we have to understand that there are basically two kinds of data within the method, the primary component data and the secondary component data.

#### 6.1 Primary components data

The primary component data is from the CPU's, I/O (also network), Memory and Queues. The first three are the system resources. Within the system resource data we can find utilizations data, CPU / IO queue length data, memory swap usage data and physical CPU data (in case of virtualization), etc.

The fourth primary component data is a very important component to keep in mind for end user processes depending on asynchronously executed services within the end user process. The amount of jobs in the queues (queue length) and the response time of the queue can influence the end user process very much. The method will find out how much this influence is.

Primary component data is the most important data to collect to determine and identifying where in your architecture your bottlenecks can be found and so to which tier your optimizations effort should go.

#### 6.2 Secondary components data

The secondary components data is basically the rest of all possible data to be collected. Examples of secondary component data are: SQL statement AWR information, number of latch waits, number of java threads, etc.

#### 6.3 Basic principles

In principle the method can work with any data source but a very important rule is that all data should be able to be related by timestamps, so time synchronization is from vital importance. To be able to do this, in some cases averaging over different timestamps has to be done (aggregate data). This has as a bad side effect that variance in the data will be reduced. Although this is the case, the relation between the metrics and the response time will not be reduced in such a way that relations between metrics will be changed.

All the collected data should be stored in one database. For this white paper there has been worked with the Oracle database, but in base also another database could be used. The advantage of using the Oracle database is the fact that Data Mining (ODM) is part of the Oracle database. Although this is the case there might be alternatives in the market to do the actual data mining (e.g. Project-R, Mahout (Hadoop), etc.).

#### 6.4 Data sources to be used

The data sources to be used can be:

- Directly from the individual servers gathered sar files, vmstat files, etc.
- Response time data from tools like RUEI, loadrunner, APM tools, etc.
- Further more the in the introduction mentioned data sources.

An important data source can be Oracle Enterprise Manager or equivalent. These kinds of tools are using agents and gather a lot of primary and secondary component data, "Method-GAPP" can use as input. The big advantages are the fact that the synchronization and gather steps are much easier. Inside the Oracle Enterprise Repository you can use the tables sysman.mgmt\$metric\_details and sysman.mgmt\$metric\_hourly to get your data. Make sure for the data you want to gather that the sampling frequency is similar with the AWR sampling frequency, this will make the data synchronization step a lot easier.

#### **6.5** Values Examination

Before doing anything it is important to check if the data is valid and in which format the data is collected. Sometimes we can find strange values like negative utilizations or utilizations far above 100%. In these cases it is important that strange values should be deleted. Utilizations over 100% can sometimes be present when the utilization is from a CPU resource within a virtual machine. The metric should in this case be treated as a secondary component. Of course utilizations are sometimes in percentages or in ratios (0-1). It is always good to plot the data in a graph to see how the data looks like. Most of the time this is the best way to identify very strange values in your data.

Next to the above it is handy to checkout if data is in multiples available. This can be encountered when looking at I/O devices, different devices have exact the same metrics reported. In the analyses just one should be chosen. The Data Mining step to determine the importance of factors (components) will also help to identify this data.

# 7 "Method-GAPP" Data Synchronization

When all the data is inside the database we try to aggregate the data on such a level that we can synchronize them based on the timestamp. It is important to check the time differences between the servers you collected the data from in the first place. Based on this you can decide to aggregate the data on a higher level, so e.g. from 10 minute interval to 30 minute interval. The end result should be one big table (or view) containing one column to identify the case (time, id), one column with the response time (R) to be analyzed (or multiples when used for several application business processes), and for each collected component one column.

#### 7.1 Workload profiles identification

When the data synchronization is done it is good to make sure it is easy to recognize what kind of workload was on the architecture of the direct involved application (chain). For example a system is totally differently used during the night than during the day when the most OLTP is done. This influence will also be discussed in the data modeling steps. One of the most important things of a drastically changing workload profile is the fact that I/O is differently cached. The problem is that e.g. instead of doing 100 I/O's for the end-user process we have to do now 1000 I/O's for the same end-user process. This is for modeling a nightmare, but we can correct our models on this by adding a metric (secondary component) about the SGA size, the number of sort processes, etc. Also by just having longer periods of data will make the model better. A start of a system is due to this fact also hard to be modeled.

# 8 "Method-GAPP" Data Mining - Explain

Now we have all the data available in one database we need a way to determine which components are of importance to predict the end user response time (R) of the investigated business process. In real big systems it is not possible to checkout every component and how it is influencing the response time of the end-user process. For this we can use data mining, to make the computer do the monks job in principle. One of the Data Mining's functions around is the attribute (predictors) importance explaining, which is in base a factorial analysis. Within Oracle Data Mining there is a procedure "EXPLAIN" within the package "DBMS\_PREDICTIVE\_ANALYTICS" (*Oracle 2011*) to do this. For this can also be found alternatives, but in this current white paper only the Oracle Data Mining's procedures have been used.

#### 8.1 Some simple tests with Swingbench

To illustrate the method some better a few simple tests were done using two different laptops (Windows 7 and Mac), with a virtual machine running Linux. To generate some load and have an end-user process, the load generator "Swingbench" (Order Entry), from Dominic Giles (Oracle) was used in character base. See Illustration 2 with the GUI, with the same input values. On Linux sar data was collected.

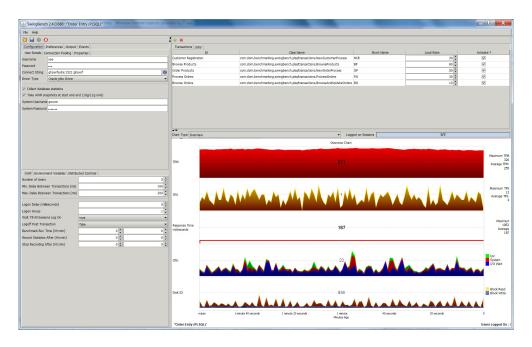
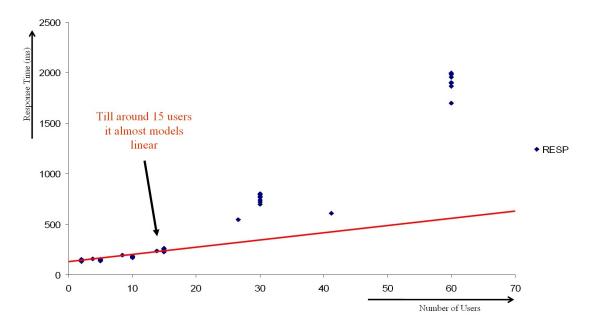


Illustration 2: The illustration shows the GUI from Swingbench, while running the load test.

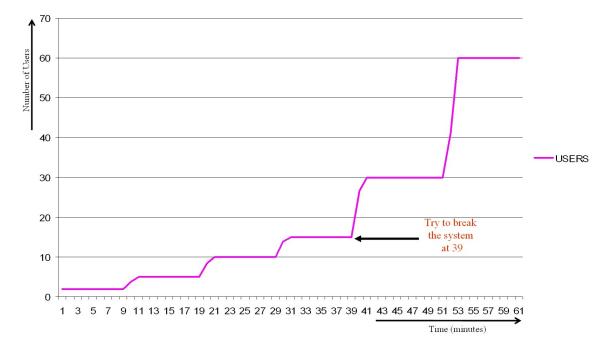
#### 8.1.1 First Swingbench Test (on Windows 7)

The test was done with different amounts of users and in this test was determined where the system looked to be overloaded. This point was around 15 users; the graph of the test is shown in graph 2.



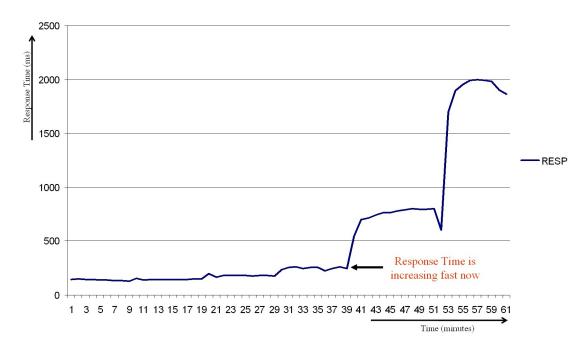
Graph 2: Graph of the measured response time (RESP) versus the number of user added in the test. While adding extra users to the test the response time suddenly started to increase rapidly above 15 added users.

The graph shows clearly how the response time at a certain point is diverting from an almost linear curve. After the above was found a new test was done now knowing that after around 15 users the response time would start to go up fast. The number of users in this new test is shown in graph 3.



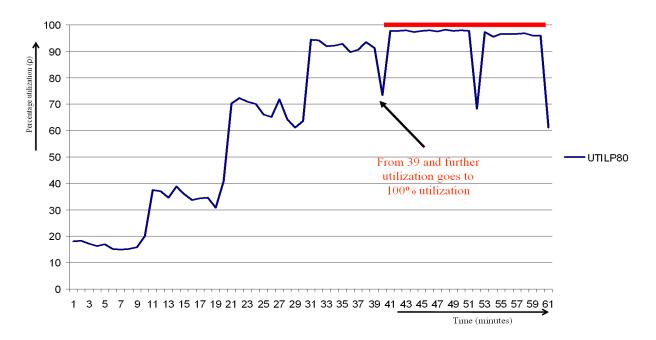
Graph 3: Graph of the measured response time (RESP) versus the time elapsed during the test. From time 39 and onwards we have tried to break the system by overloading it with too many users.

Although it was already clear the system would get overloaded the response time through time was measured with increasing amount of users. This behavior is illustrated in the graph of graph 4.



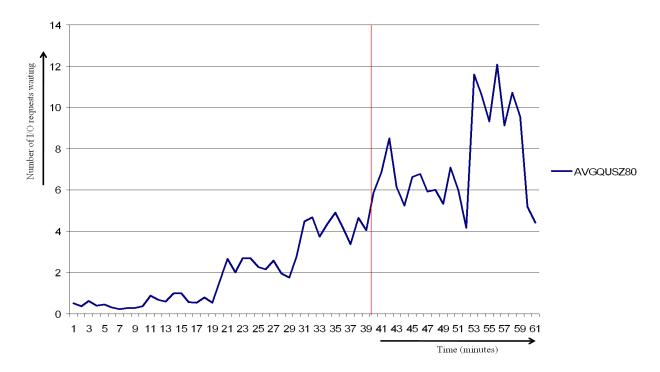
Graph 4: Graph of the measured response time (RESP) versus the time elapsed during the test. From time 39 and onwards we have tried to break the system by overloading it with too many users.

Looking at what is going on at the background we can see for the utilization of the I/O (Device 8-0 in the test), the graph shown in graph 5.



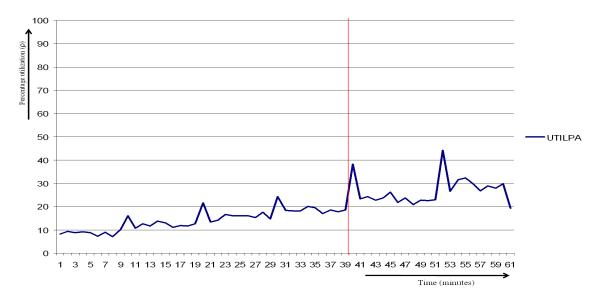
Graph 5: Graph of the I/O utilization of device 8-0 (AVGQUSZ80) versus the time elapsed during the test. The horizontal red line shows how the utilization is starting to hit the 100% utilization.

The graph shows clearly how we start to hit the 100% utilization and due to that probably start to queue on the I/O device. You can see how the run queue of the I/O device is getting higher in the graph of graph 6.



Graph 6: Graph of the average queue length for device 8-0 (AVGQUSZ80) versus the time elapsed during the test. The vertical red line shows the time in the test we tried to break the system by overloading it with too many users.

If we look at the CPU utilization during the test we can find the graph shown in graph 7.



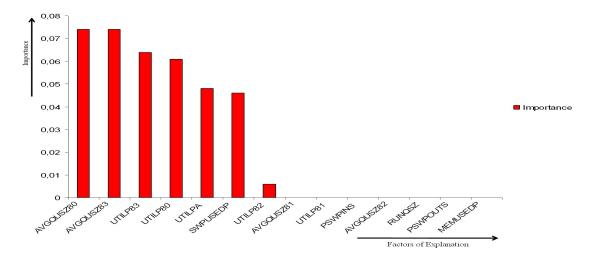
Graph 7: Graph of the CPU utilization (UTILPA) versus the time elapsed during the test. The vertical red line shows the time in the test we tried to break the system by overloading it with too many users.

The graphs show clearly that the system is fully I/O bound and that the I/O is probably mostly responsible for the measured response time.

#### 8.2 Using the Factorial Analysis on Primary components

During the test on the windows 7 laptop it became already clear that the I/O looked to be the bottleneck during the test. To check, the factorial analysis was done, by the "EXPLAIN" procedure to

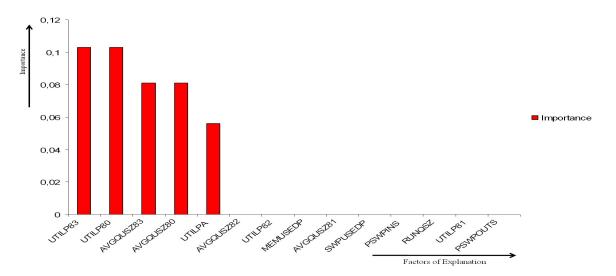
determine which components are actually the most important in predicting the measured end-user response time has been used and the graph of this factorial analysis is shown in graph 8.



Graph 8: Graph shows the factorial analyses (GAPP primary components) of the measured (full 61 minutes) response time in the test. In the graph in order: average queue length of device 8-0 (AVGQUSZ80), average queue length of device 8-3 (AVGQUSZ83), utilization of device 8-3 (UTILP83), utilization of device 8-0 (UTILP80), utilization CPU's (UTILPA), swap used percentage (SWPUSEDP), utilization of device 8-2 (UTILP82), average queue length of device 8-1 (AVGQUSZ81), utilization of device 8-1 (UTILP81), page file swapins (PSWPINS), average queue length of device 8-2 (AVGQUSZ82), run queue of CPU (RUNQSZ), page file swapouts (PSWPOUTS) and memory used percentage (MEMUSEDP).

The graph shows clearly that the run queue of device 8-0 and device 8-3 are the most important predictors, after that the utilization of device 8-3 and device 8-0. A small investigation shows that Device 8-0 and 8-3 are in real the same device, so only device 8-0 is further used as the I/O device.

If we only do a factorial analysis on the data till the point the system gets overloaded we can find the graph in graph 9.



Graph 9: Graph shows the factorial analyses (GAPP primary components) of the measured (first 39 minutes) response time in the test. In the graph in order: utilization of device 8-3 (UTILP83), utilization of device 8-0 (UTILP80), average queue length of device 8-3 (AVGQUSZ83), average queue length of device 8-0 (AVGQUSZ80), utilization CPU's (UTILPA), average queue length of device 8-2 (AVGQUSZ82), utilization of device 8-2 (UTILP82), memory used percentage (MEMUSEDP), average queue length of device 8-1 (AVGQUSZ81), swap used percentage (SWPUSEDP), page file swapins (PSWPINS), run queue of CPU (RUNQSZ), utilization of device 8-1 (UTILP81) and page file swapouts (PSWPOUTS).

In the graph it is clear that now the utilizations are more important as the I/O queues, and that the CPU utilization has also some influence on the end-user response time.

Basically the factorial analyses can determine which component is the most responsible for the variance (attribute importance) in the business (end-user) process response time. In this way we can find out in very complex systems where the response time is the most influenced. To make in bigger systems with a lot of components the diagrams a little easier to interpret we can give the component with the highest importance the index number 100, and all other component on ratio a lower index number.

If we checkout now the diagrams (1 and 3) from the beginning of the whitepaper what the factorial analyses actually is doing we can look at the diagram in Diagram 4.

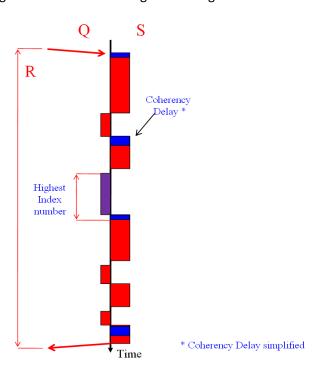
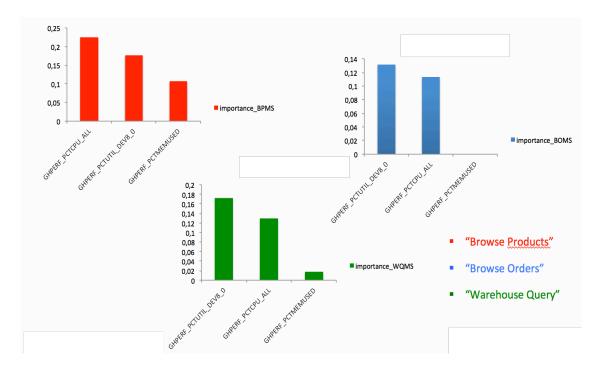


Diagram 4: Diagram how the factorial analyses graphs are mapped to the sequence diagram. The biggest variance of queuing time (Q) and coherency together for a specific part of the architecture will have the highest bar in an importance graph (index number 100% or the highest importance).

The diagram makes clear that basically the factorial analyses find the performance bottlenecks in the system. If also components are involved outside the "chain" we can find out what component, maybe not even part of the "chain" have influence on our investigated end-user process. Now we know the bottlenecks we can start tuning these parts.

#### 8.2.1 Second Swingbench Test (on Mac)

To illustrate better the power of the factorial analyses, a very similar test as the first Swingbench test was performed on a Mac. The Mac had an SSD drive inside what made the response time decrease a lot. The data gathered from this test was not done on the average of all the different transactions started in Swingbench (unweight average), but were gathered from each individual transaction ran in the test. The results of three of the transactions and their factorial analyses are shown in graph 10.



Graph 10: Graphs show the factorial analyses (GAPP primary components) of three different transactions (response time in ms) from the second test (Mac). The graphs make clear that each transaction is differently hit by the different components, although the transactions where running simultaneous. In the graphs: utilization of device 8-0 (GHPERF\_PCTUTIL\_DEV8\_0), utilization CPU's (GHPERF\_PCTCPU\_ALL), memory used percentage (GHPERF\_PCTMEMUSED).

In the graphs it becomes very clear how different transactions, which were run in Swingbench simultaneous, are influenced by the different components in the technical architecture differently. This is exactly as shown in diagram 1 (Method-R). So the factorial analysis shows us exactly where the most variance in our response time of a specific transaction is coming from. So for example the "Browse Products" (BP) transaction is more CPU bound, while the other two transactions "Browse Orders" (BO) and "Warehouse Query" are more I/O bound. Of the last two the "Warehouse Query" (WQ) is the most I/O bound.

#### 8.3 Important rules regarding component choosing

When choosing the components for the "EXPLAIN" procedure it is important to choose components, which are as less as, possible correlated. This means normal to have just one primary component per resource. In the case of the two tests the utilization of device 8-0 and the CPU utilization were chosen.

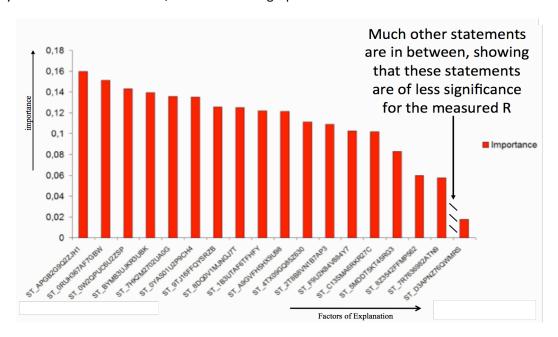
One of the problems with data mining in general is the fact that predictors should be independent. A lot of data in the world has some correlation between each other and for this problem Ridge Linear regression can help. By playing with the Ridge variable sometimes better linear models can be found.

#### 8.4 Using the Factorial Analysis on Secondary components

The factorial analyses can also be done on secondary components. This can be very powerful for example to find out, which SQL statements, events, etc. are involved in our investigated end-user process.

#### 8.4.1 SQL Statement response time

If we go back to our Swingbench tests and we use elapsed\_time\_delta / executions\_delta from dba\_hist\_sqlstat to obtain the SQL statement response time than we are able to make a factorial analyses based on the first test, this is shown in graph 11.



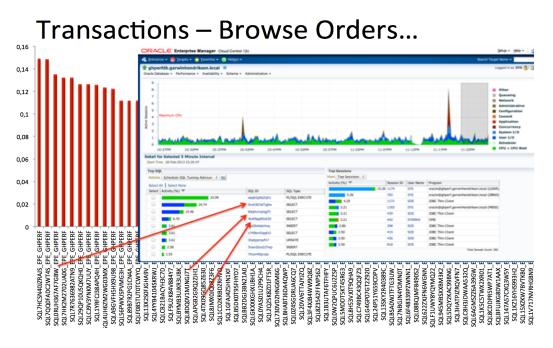
Graph 11: Graph shows the factorial analyses (GAPP secondary components) of the involved SQL statements in the first Swingbench test (Windows 7). The analysis was done to explain the unweight average response time of some transactions in Swingbench.

The graph shows that the SQL statements on the left of the graph are very important to predict the end user response time. The statements on the right are not of significant importance. It becomes even clearer knowing what the statements are, see list 1.

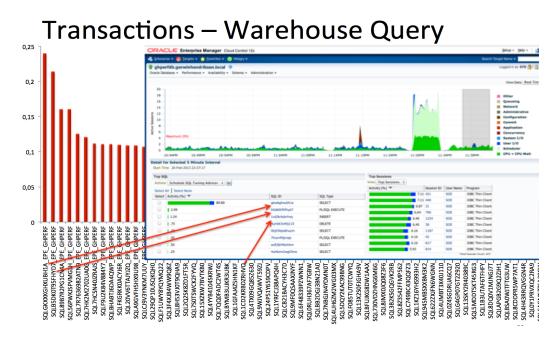
- 'apgb2g9q2zjh1'
  - BEGIN:1:= orderentry.browseandupdateorders(:2,:3,:4); END;
- '0ruh367af7gbw'
  - SELECT ORDER\_ID, ORDER\_MODE, CUSTOMER\_ID, ORDER\_STATUS, ORDER\_TOTAL,
    SALES\_REP\_ID, PROMOTION\_ID FROM ORDERS WHERE CUSTOMER\_ID = :B2 AND ROWNUM
- '0w2qpuc6u2zsp'
  - BEGIN :1 := orderentry.neworder(:2 ,:3 ,:4 ); END;
- 'bymb3ujkr3ubk'
  - INSERT INTO ORDERS(ORDER\_ID, ORDER\_DATE, CUSTOMER\_ID, WAREHOUSE\_ID) VALUES (ORDERS\_SEQ.NEXTVAL + :B3 , SYSTIMESTAMP , :B2 , :B1 ) RETURNING ORDER\_ID INTO :O0
- '7hk2m2702ua0g'
  - WITH NEED\_TO\_PROCESS AS (SELECT ORDER\_ID, CUSTOMER\_ID FROM ORDERS WHERE ORDER\_STATUS <= 4 AND WAREHOUSE\_ID = :B1 AND ROWNUM < 10 ) ...</li>
- '0yas01u2p9ch4'
  - INSERT INTO ORDER\_ITEMS(ORDER\_ID, LINE\_ITEM\_ID, PRODUCT\_ID, UNIT\_PRICE, QUANTITY)
    VALUES (:B4 , :B3 , :B2 , :B1 , 1)
- Etc.
- Not significant statement: 'd3apnz76qwmrs'
  - select pol#, usr\_name, usr\_labels, package, privs from lbac\$user\_logon where usr\_name
    username

List 1: List with SQL statements involved in the Swingbench tests.

If we look at our second test again where we investigated the transactions ran simultaneously and we do some factorial analyses on them to explain the different transaction response times, we are able to find for each individual transaction the most important SQL statements. In graphs 12 and 13 is shown how the factorial analyses for a specific transaction, while all transactions where running simultaneously, can be mapped on Top Activity SQL statements from Oracle Enterprise Manager 12C while running only that specific transaction.



Graph 12: Graph shows the factorial analyses (GAPP secondary components) of the involved SQL statements in the second Swingbench test (Mac). The analysis was done to explain response time of the "Browse Orders" transaction. The arrows indicate how the factorial analysis maps on the found top activity queries when running only the "Browse Orders" transaction.

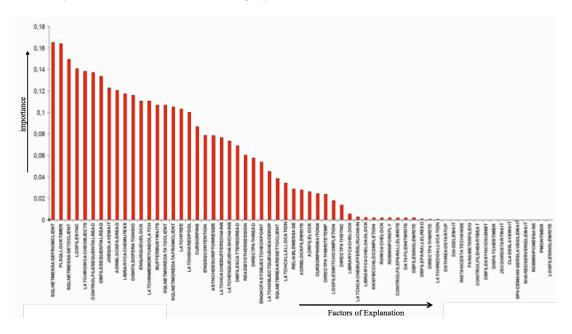


Graph 13: Graph shows the factorial analyses (GAPP secondary components) of the involved SQL statements in the second Swingbench test (Mac). The analysis was done to explain response time of the "Warehouse Query" transaction. The arrows indicate how the factorial analysis maps on the found top activity queries when running only the "Warehouse Query" transaction.

The graphs show that the factorial analysis can determine the involved SQL statements in a specific transaction if they are running simultaneously. The fact that the top activity queries are not all in the top of the factorial analyses is because the factorial analysis shows the SQL statements, which are the most responsible for the variance in response time and not what is the most time consuming query. So the top query not so important for the factorial analysis is basically performing very constantly (SQL statement response time with little variance).

#### **8.4.2** Events

In a very similar way as described for the SQL Statements, the events also can be analyzed. The factorial analysis for the events is shown in graph 14.



Graph 14: Graph shows the factorial analyses (GAPP secondary components) of the involved events in the first Swingbench test (Windows 7). The analysis was done to explain the unweight average response time of some transactions in Swingbench.

The outcome of the factorial analysis as shown in the graph is very similar as found in the AWR report, see illustration 3 (except for the non waiting events in the analysis).

- SQL\*Net message from client
- PL/SQL lock timer
- SQL\*Net message to client
- log file sync
- latch: row cache objects
- control file sequential read
- db file sequential read
- Etc.

Top 5 Timed Foreground Events

Event	Waits	Time(s)	Avg wait (ms)	% DB time	Wait Class
log file sync	12,599	972	77	59.54	Commit
db file sequential read	29,465	379	13	23.19	User I/O
DB CPU		209		12.82	
resmgr:cpu quantum	2,475	15	6	0.91	Scheduler
log file switch (private strand flush incomplete)	28	14	509	0.87	Configuration

The data used in Method-GAPP was over a lot of hours the AWR report shown only over a 10 minute time slice.

Event	Waits	%Time -outs	Total Wait Time (s)	Avg wait (ms)	Waits /txn	% DB time
log file sync	12,599	0	972	77	0.43	59.54
db file sequential read	29,465	0	379	13	1.00	23.19
resmgr:cpu quantum	2,475	0	15	6	0.08	0.91
log file switch (private strand flush incomplete)	28	0	14	509	0.00	0.87
buffer busy waits	37	0	5	139	0.00	0.31
read by other session	10	0	0	33	0.00	0.02
SQL*Net message to client	19,922	0	0	0	0.68	0.02
library cache: mutex X	124	0	0	2	0.00	0.01
ADR block file read	4	0	0	38	0.00	0.01
control file sequential read	1,742	0	0	0	0.06	0.01
log file switch completion	2	0	0	30	0.00	0.00
latch free	11	0	0	4	0.00	0.00

Illustration 3: Illustration shows how the events from the factorial analysis can be mapped on an AWR report.

# 9 "Method-GAPP" Data Modeling

As many of you will realize is the fact that for example CPU utilization and I/O utilization will not have a linear correlation with response time. The correlation curve behind is dependent on the number of server / threads of the resource. In the past Erlang made a formula, the Erlang-C formula, to calculate the probability that all the servers (e.g. CPU's) for the resource are busy (*Gunther 2005*). Together with the formula for response time: R=Q+S we are able to calculate the response time (R), for given number of server / threads, see formula 1.

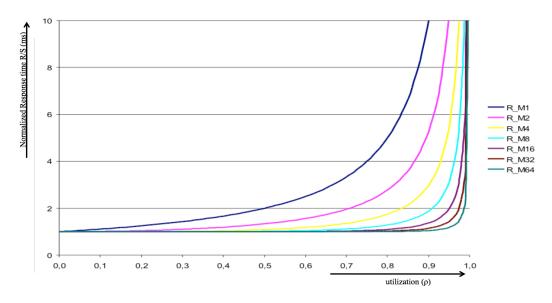
$$C(m,\rho) = \frac{\frac{(m\rho)^m}{m!}}{(1-\rho)\sum_{n=0}^{m-1} \frac{(m\rho)^n}{n!} + \frac{(m\rho)^m}{m!}} \qquad R = \frac{C(m,\rho)S}{m(1-\rho)} + S$$

Normalized R for M1 curve:

$$R = S(\frac{C(m,\rho)1}{m(1-\rho)} + 1) \qquad \frac{/}{S} \qquad R_M = 1(\frac{C(1,\rho)1}{1(1-\rho)} + 1)$$

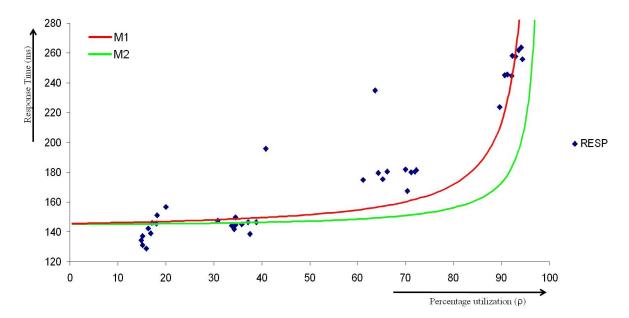
Formula 1: The first formula (1) (from left to right) is to calculate the probability (C) that all the servers (e.g. CPU's) for the resource are busy, with as input: number of servers / threads (m) of the resource (e.g. number of CPU's) and utilization of the resource (p). The formula was created by Erlang and is called the Erlang-C formula. The second formula (2) is to calculate the resulting response time (R) when the probability (C), the number of servers / threads (m) and service time (S) is filled in. The third formula (3) is derived from the second formula (2) and the fourth formula (4) is formula (3) divided by service time (S) and for the number of threads is "1" (M1) filled in. So formula (4) is describing the so-called normalized M1 curve.

When we use the formulas to calculate the normalized M curves you can find graph 15.



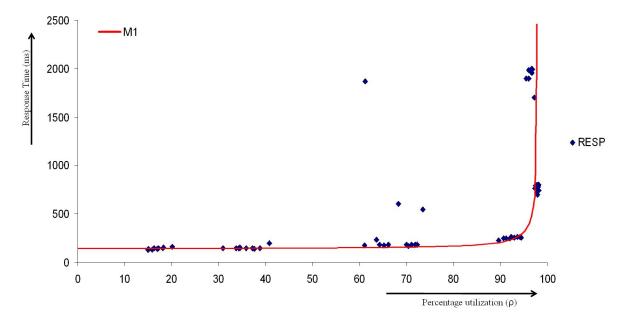
Graph 15: The graph shows how with different number of servers / threads, the normalized (brought back to 1 ms) response time (R\_Mx) would be predicted based on Erlangs-C formula, with given utilization (ρ).

If we go back to the first Swingbench test and when plotting the response time data against the I/O device utilization with the M1 and M2 modeled curves we can find the graph in graph 16.



Graph 16: Graph of the measured response time (RESP) versus the utilization of the I/O device (device 8-0 in the test). The M1 and M2 curves show how the data is fitting to the graphs. Important to remark is the fact that response time is also influenced by the CPU impact, coherency and other unknown components (e.g. the Swingbench client) what makes the data not fit complete.

In the graph it is clear that the influence of the I/O is probably described as something like an M1 curve. If we try the same fit with the data of the overloaded situation we can find the curve in graph 17.



Graph 17: Graph of the measured response time (RESP) versus the utilization of the I/O device (device 8-0 in the test). The M1 curve drawn in the graph shows how the data is fitting the M1 curve.

The graph shows clearly that the influence of the I/O device is far from linear and that the M1 curve is highly likely to be correct.

## 10 "Method-GAPP" Data Mining - Model

After knowing what the most important components are we can actually start making a model of the data to predict the measured response time. Within Oracle Data Mining we can use function "Regression" with algorithm "Linear Regression (GLM)", these functions are also available in other data mining's tools. We use this function on a view with only the selected components. With a small trick we can in this way also determine which M-curve is fitting the best with the data. Inside Oracle we can use the Oracle Data Miner, the latest Oracle SQL Developer or from SQL the package "DBMS\_DATA\_MINING" (Oracle 2011).

To determine the best M-curve fitting we can use an iterative process. We start with a view containing all the selected columns as the raw data retrieved from the system, and we calculate a model with the involved columns, after the calculation we store the "root squared error" from the model, and do the calculation again with a normalized response time of the most important factor with the M1 curve, with M2 curve, etc. After some calculation we than can find what the smallest "root squared error" is and so determine which M-curve was fitting the best. We repeat the procedure with the second most important component with the already determined most important component M-curve in place. After all components have been done the procedure is repeated again, starting with the most important component. In illustration 4 is illustrated what the M-curve fitting tries to accomplish.

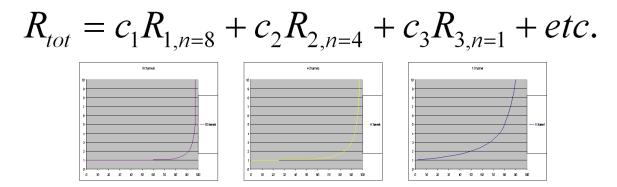


Illustration 4: The picture shows an example of what the M-curve fitting tries to accomplish. In the picture the "c" is the coefficient of the calculated model and the  $R_1$ ,  $R_2$ , etc. is the normalized response time of the "n" amount of servers, for that specific component.

After we know the best M-curve fit for each component we can calculate the model with the components transformed with the correct M-curve. See the picture in illustration 5 for a calculated model (first Swingbench test), after the M-curves have been determined for the I/O and CPU.

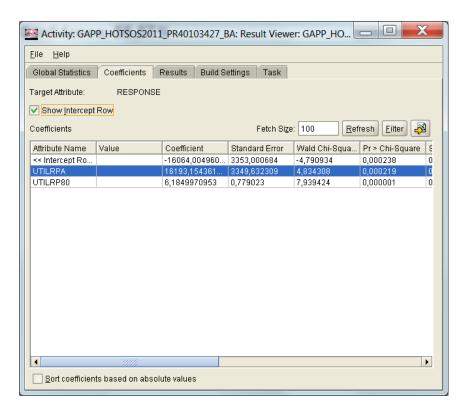


Illustration 5: Picture shows the result of the data mining done in Oracle Data Miner on the selected primary components (first Swingbench test).

The discussed procedure delivers us via the "coefficients" (see illustration 5) of the model for the predictors and the "intercept row" of the model, the actual linear formula's as shown in formula 2.

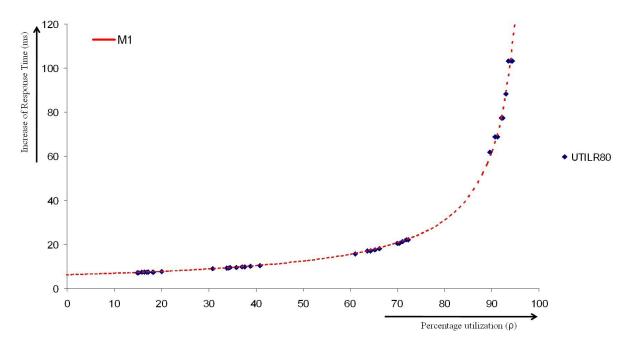
$$\begin{split} R_{RESP} &\approx 16193.15 \times M_{CPU\,n=4} + 6.19 \times M_{IO\,n=1} + -16064.00 \\ R_{RESP} &\approx ((16193.15 \times M_{CPU\,n=4}) - 16193.15) + 6.19 \times M_{IO\,n=1} + (-16064.00 + 16193.15) \\ R_{RESP} &\approx ((16193.15 \times M_{CPU\,n=4}) - 16193.15) + 6.19 \times M_{IO\,n=1} + 129.15 \end{split}$$

$$R_{RESP} \approx R_{UTILRAU} + R_{UTILR80} + R_{REST}$$

Formula 2: Formula extracted of the linear regression model (first Swingbench test) used for the data mining. In this formula the modeled CPU utilization (Mcpu with 4 servers, normalized with Erlang-C) and the modeled I/O utilization (Mio with 1 server, normalized with Erlang-C) was used to calculate the predicted response time (Rresp). At the bottom we can say that the response time from the CPU component (Rutilrau), the response time of the IO component (Rutilr80) and the always present rest response time (Rrest) is giving us the predicted overall response time (Rresp).

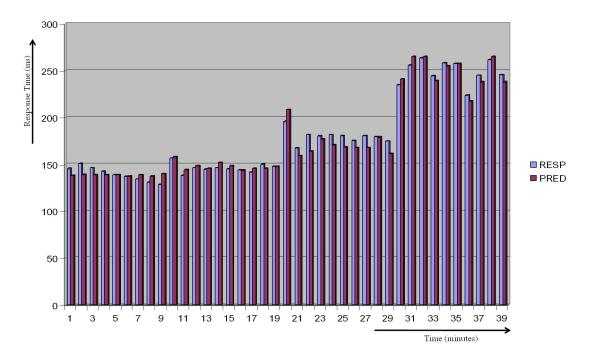
The formula makes clear that on base, when no resources are used (steady state) the average response time of 135,34 ms (0+6,19+129,15) can be found. The 135,34 ms consist of the different component service times and the unaccounted time. Important to realize is the fact that big change in I/O caching can make the modeling unreliable (see Workload profiles). Another remark regarding the workload profiles is the fact that on occasion when long running processes are running on the system (e.g. night batch), a resource might not be modeled e.g. via a M4 curve, but e.g. with a M1 curve. This can give big issues with the found formula. So it might be an option to split the data in different workload profiles (e.g. day and night).

Now we know the formula behind the model we can checkout how according the model the I/O is making the end-user response time to increase, this is plotted in graph 18.



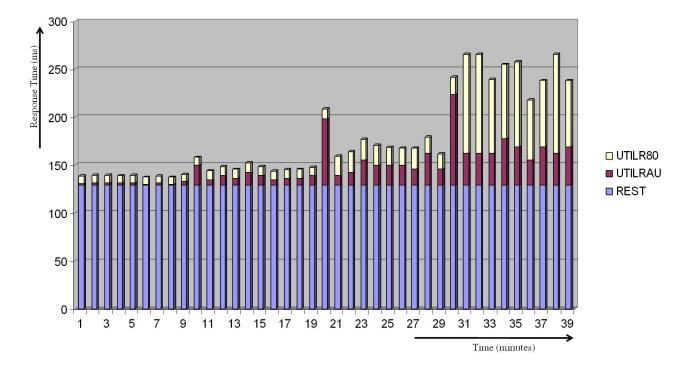
Graph 18: Graph, which shows how, according the calculated model the I/O (UTILR80) versus the utilization of the I/O device 8-0 in the first Swingbench test is influencing the end-user response time.

Now we have the model for the first Swingbench test (Windows 7), we should check how the found formula is actual describing the reality. A bar graph representation can be found in graph 19.



Graph 19: Graph of the measured response time (RESP) and the predicted response time (PRED) by the model based on CPU and I/O versus the time elapsed during the first Swingbech test (Windows 7).

We are now also able to create a graph with each of the components influence on the total predicted response time. This is shown in the graph 20.



Graph 20: Graph of the predicted response time by the model based on CPU and I/O versus the time elapsed during the first Swingbench test. In the bars the CPU component (UTILRAU), the I/O component of device 8-0 in the test (UTILR80) and the base response time (REST) is shown.

The graph shows clearly what the influence is from each individual component within the response time. Remarkable is the bar at time 30 minutes, the influence of the CPU is here very high but the impact of the I/O is little.

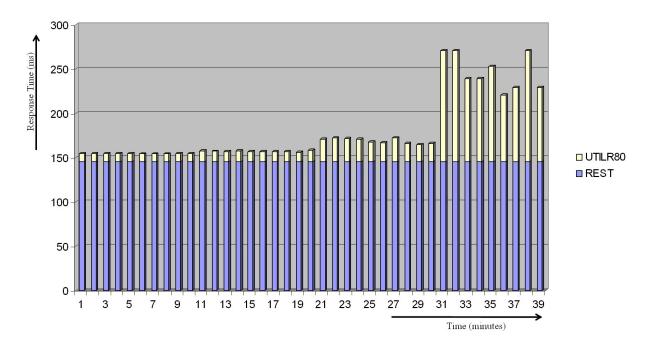
To have a good understanding of the fact what component selection can have as impact, a model was created without the CPU, so only based on the I/O. The formula for this model is shown in formula 3.

$$R_{RESP} \approx 7.52 \times M_{IO_{n=1}} + 145.41$$

$$R_{RESP} pprox R_{UTILR80} + R_{REST}$$

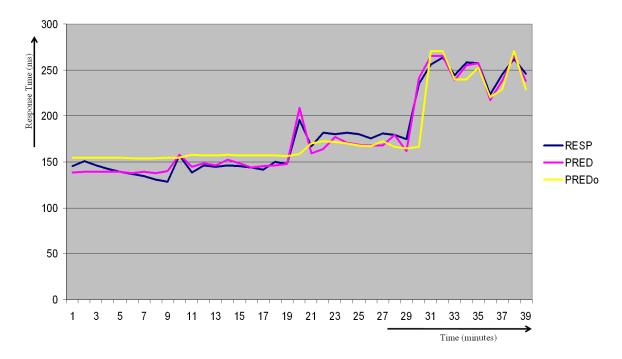
Formula 3: Formula extracted of the linear regression model (first Swingbench test) used for the data mining. In this formula the modeled I/O utilization (Mio with 1 server, normalized with Erlang-C) was used to calculate the predicted response time (Rresp). At the bottom we can say that the response time of the IO component (Rutilr80) and the always present rest response time (Rrest) is giving us the predicted overall response time (Rresp).

After having this formula, we are now again able to create a graph with each of the components influence on the total predicted response time. This is shown in the graph 21.



Graph 21: Graph of the predicted response time by the model only based on I/O versus the time elapsed during the first Swingbench test. In the bars the I/O component of device 8-0 in the test (UTILR80) and the base response time (REST) is shown.

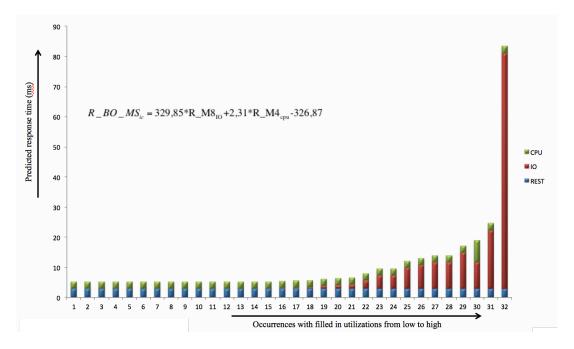
The figure shows again a nice bar chart, explaining where the response time was used. But if we look now at the data at minute 30 we can't see the actual high response time (see graph 20). Now that we have the two created models we can compare them visual versus the actual measured response times. This is shown in the graph 22.



Graph 22: Graph of the measured response time (RESP), the prediction of the model with CPU and I/O (PRED) and the prediction of the model only based on I/O (PREDo) versus the time elapsed during the first Swingbench test.

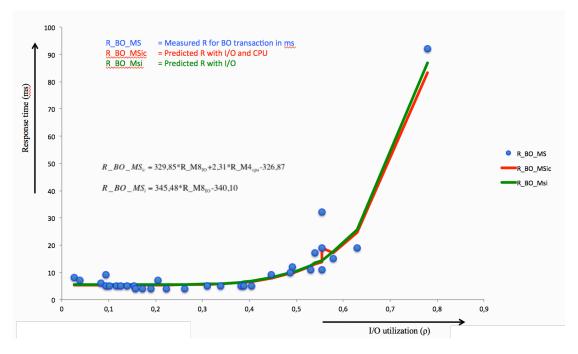
The graph shows clearly that the created model including the CPU has a better predicting value than the one created without the CPU.

As we have done for the first Swingbench test we can also make the breakdown for the second Swingbench test (Mac). The breakdown is shown in graph 23.



Graph 23: Graph of the predicted response time of the "Browse Orders" (BO) transaction by the model based on I/O and CPU versus the different utilizations (from low to high) during the second Swingbench test. In the bars the I/O component (IO), CPU component (CPU) and the base response time (REST) is shown.

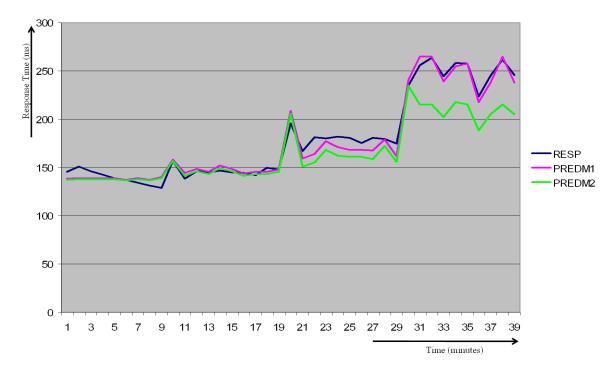
The graph is comparable with the first test graph 21. If we compare different models and checkout the real measured response time in the second Swingbench test we can find graph 24, which looks also comparable as found in the first test in graph 22.



Graph 24: Graph of the measured response time (blue dots) of the "Browse Orders" (BO) transaction, the prediction of the model with CPU and I/O (R\_BO\_MSic) and the prediction of the model only based on I/O (R\_BO\_MSi) versus the utilization of the I/O during the second Swingbeach test.

## 11 "Method-GAPP" Data Mining - Predict

Now we have been able to make models of measured response times, we will go a step further by using the data mining models to predict what will happen if things are changing in the infrastructure. Graph 25 shows the prediction of the response time in case the number of I/O channels was changed from 1 to 2 (if it is possible) and the utilization of the I/O stays the same with the same amount of transactions. In real we will see that a positive I/O configuration change, will allow that the same amount of users will do more transactions and due to this fact increase also the CPU utilization, so the graph is a good indication of what will happen in real.



Graph 25: Graph of the measured response time (RESP), the prediction of the model with an M1 modeling of the I/O (PREDM1) and the prediction of the model with an M2 modeling of the I/O (PREDM2) versus the time elapsed during the test.

The graph shows well how the response time would be much lower in the area's the utilization gets higher. Of course normally double the amount of channels will also reduce the utilization of the device. So the effect will be bigger than shown in the graph. Based on the graph we can decide if the change would be worth the investment.

In the coming paragraphs we will look deeper into real customer cases where "Method-GAPP" was used very successful. Because of the fact that the method was not as far as it is now, the analyses was done using a function inside Oracle Data Mining package "DBMS\_PREDICTIVE\_ANALYTICS", called "PREDICT" (*Oracle 2011*). Still this function might be very handy if the data is very complex and hard to be modeled with "linear regression". So it is still a very valuable alternative of the "linear regression" model. The big advantage of having a linear model over a model created with the "PREDICT" procedure is the fact that basically we can see what the impact of each individual component is, by checking out the terms in the linear equation.

## 12 "Method-GAPP" Data Interpretation

When starting to predict response times and influences of technical changes, the "Method-GAPP" analyses are very helpful but still it is important to use your performance and subject matter expertise to make a correct interpretation of the analyses. The interpretation of the analyses is a vital step in the whole method.

For example as the secondary component "network packets" is selected in a factorial analyses it might be very well possible that it is of high importance, but in real it doesn't mean that we have a problem with the network. It only says that when we are using the system more we sent more network packets through the system. This kind of interpretation problems can occur on all kind of levels. To ensure the interpretation is correct we should base most conclusions on the primary component factorial analyses. When using the secondary components we should always be more careful.

As a rule of thumb when doing a factorial analyses of the primary components we can say a higher importance of the queue length than of the utilization is a strong indication that the involved component is a performance bottleneck (we can also use the formula's of the correlation of queue length vs. the response time, as we have done for utilization vs. the response time). This was also the indication when we overloaded our system in the small test with "Swingbench" we did. Still we need to be careful because for some systems 100% utilization is not really a problem (e.g. Netapp I/O), while on other systems 100% utilization means that the resources are starting to be exhausted.

#### 12.1 Customer Case: Banking Application

In illustration 6 a schematic representation is shown of the customer case. The problem in this environment was the fact that there was no test system and tracing was not allowed. Basically we were trying to find a performance bottleneck with both hands tied on our back.

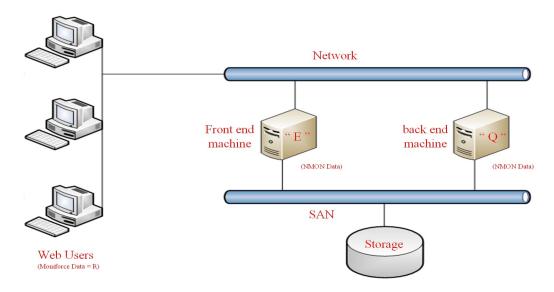
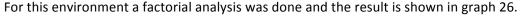
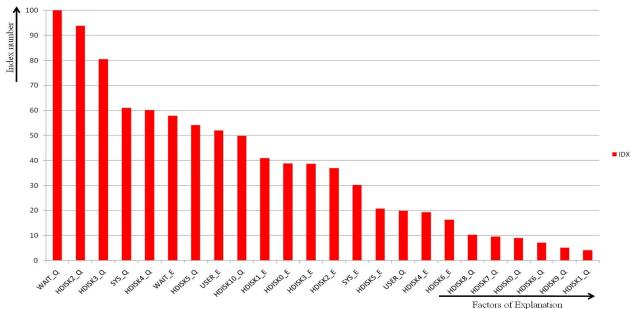


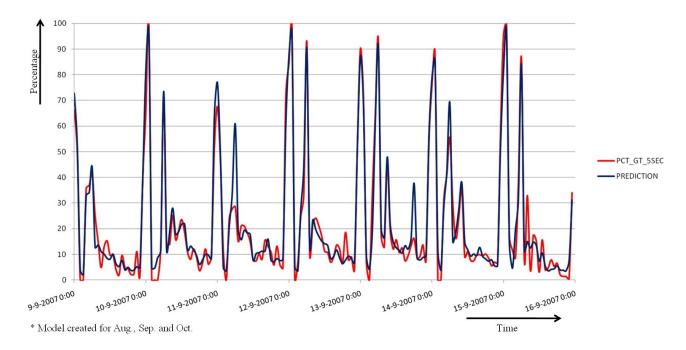
Illustration 6: Figure shows how schematic the customer technical architecture looks like. The application is a banking application, which allows users at home to check their bank account, etc. The architecture has two databases, the database on machine "E", which is the database always giving an answer, and a database on machine "Q" which refreshes "E". The business case here is that refreshing takes more than 5 seconds what will mean that the end user gets old bank account data.





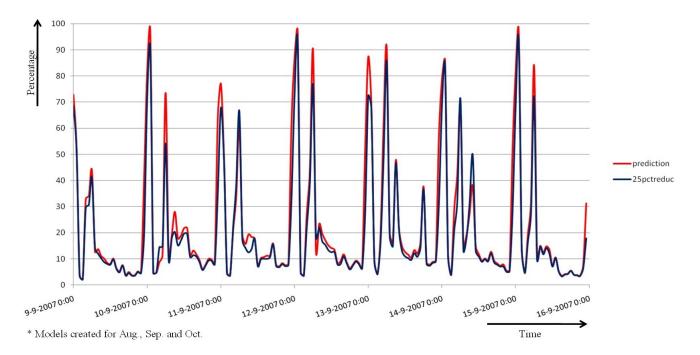
Graph 26: Graph shows the factorial analyses of the measured percentage failures of response times higher than 5 seconds. The model was based on months August, September and October 2007. In the graph in order, the first five: CPU I/O wait percentage of machine Q (WAIT\_Q), utilization of device 2 at machine Q (HDISK2\_Q), utilization of device 3 at machine Q (HDISK3\_Q), CPU system mode percentage of machine Q (SYS\_Q) and utilization of device 4 at machine Q (HDISK4\_Q).

In the graph it is clear that the first three components have the most influence on the response time. The three metrics can be used to create a model to describe the response time. In this case the earlier mentioned Oracle Data Mining "PREDICT" procedure was used to create the model. The predicted response time graph of the created model is shown in graph 27.



Graph 27: Graph of the measured percentage failures of response times higher than 5 seconds versus the predicted percentages over one week in September 2007. The model was based not only on the week presented, but also on months August, September and October 2007. Modeling was done with the dbms\_predictive\_analytics.predict routine.

In the graph it is clear that the prediction is very accurate describing the real measured response time. By having the model it becomes possible to make a copy of known cases and modify its input metrics values as being new unknown cases to predict the new response time graph. In graph 28, this prediction is plotted against the old predicted response time. In this case the I/O metrics of the copied cases were reduced with 25% and the response time for these new cases was predicted.



Graph 28: Graph of the predicted percentage failures of response times higher than 5 seconds versus the prediction when I/O was reduced with 25% over one week in September 2007. The model was based not only on the week presented, but also on months August, September and October 2007. Modeling was done with the dbms\_predictive\_analytics.predict routine.

The graph shows clearly how the new response time would look like, after reducing the I/O with 25%. Based on the outcome we can decide if the hardware investment would be worth the money. So we have an instrument now in the form of "Method-GAPP" to do predictions of the new situation before actual doing the investment.

After doing a deeper investigation of the involved hard disks in the factorial analyses the problem could be pinpointed to a wrongly setup in the backup of the system. A simple change of the involved backup procedure made the performance problems go away.

#### 12.2 Customer Case: Time and Labor Application

In this customer case a complex environment was analyzed, the system consist of one big physical machine with on top of it 15 virtual machines (LPAR's at IBM). The case here is a very good example on how not directly involved components can have a big influence on the measured response time. In illustration 7 the environment is schematically represented. In the representation the directly involved LPARS are written in red (in the application "chain").

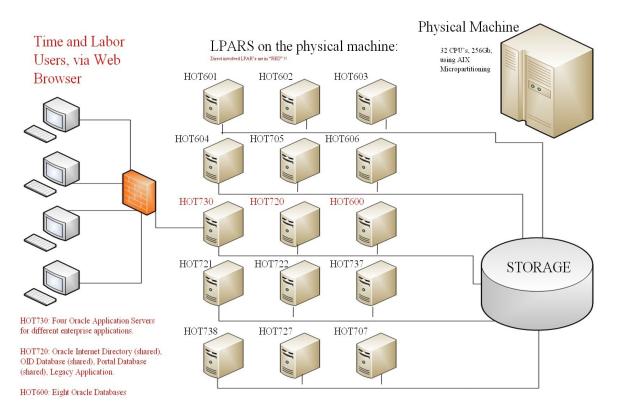
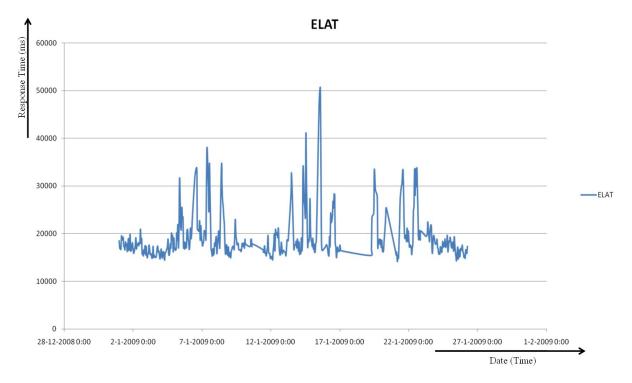


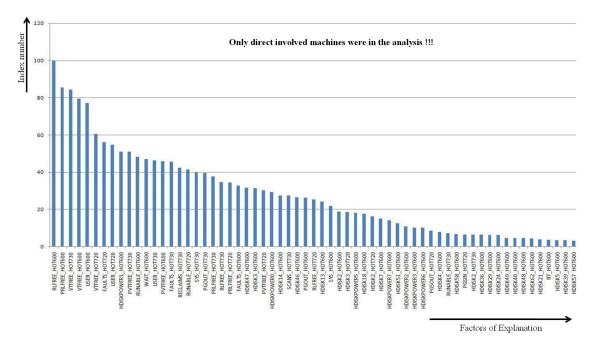
Illustration 7: Illustration shows how schematic the customer technical architecture looks like. The application is a Time and Labor application. The architecture consists of 15 LPAR's (virtual machines) on one big physical AIX machine. The LPAR's "HOT730", "HOT720" and "HOT600" are in the "chain" of the application. The other 12 LPARS are on the same physical machine, but have no functional impact in the application process (chain) analyzed.

#### The response time of the involved end-user process is shown in graph 29.



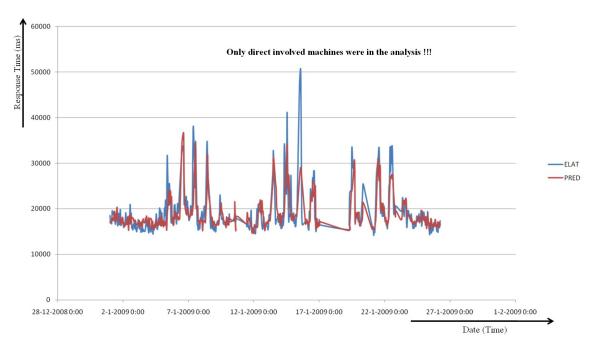
Graph 29: Graph of the measured response time (ELAT) versus the elapsed time (month January 2009).

The graph shows that there is a lot of variance in the measured response time. To get insight in the components, which are of high influence on the variance we do a factorial analysis on components of, the LPAR's direct in the "chain" of the application. The result of this factorial analysis is shown in graph 30.



Graph 30: Graph shows the factorial analyses of the measured response times. In the graph the first columns have to do with memory on different LPAR's. In this predict run only the direct involved LPAR's were in the analysis.

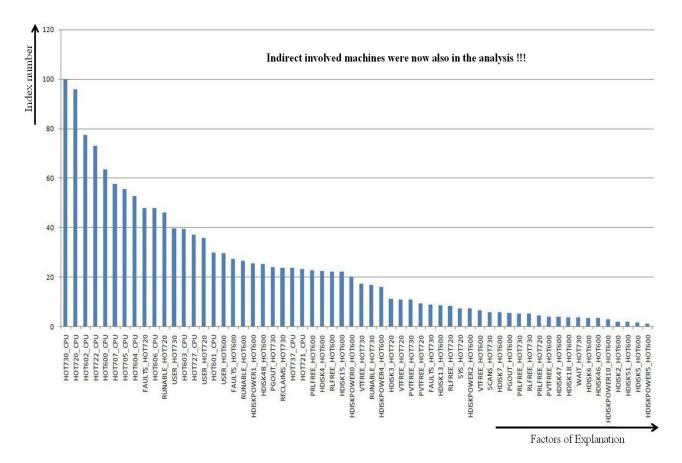
The factorial analysis shows high impact of the memory of different LPAR's in the "chain". Based on the factorial analyses we can create a model with the procedure "PREDICT". The result of this model is shown in graph 31.



Graph 31: Graph of the measured response times (ELAT) versus the predicted response time (PRED) over one month in January 2009. Modeling was done with the dbms\_predictive\_analytics.predict routine, over only the direct involved LPAR's.

The graph shows that a lot of the measured response time is predicted, but for some reason some high peeks are not predicted at all. At this time we probably don't have enough data in the model to model the measured response time well.

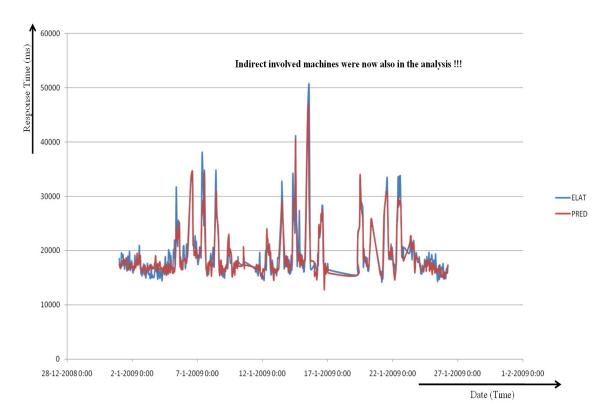
After we draw this conclusion also data from the other LPAR's on the same physical machines was collected and put also in the database. So the collected data also contained now the not directly involved LPAR's. The result of the factorial analyses is shown in graph 32.



Graph 32: Graph shows the factorial analyses of the measured response times. In the graph the first columns have to do with the physical CPU's at different LPAR's. In this predict run also the indirect involved LPAR's were in the analysis.

The factorial analyses show now a total different view on the situation. The first bars in the factorial analyses have all to do with the physical CPU amount used of not only the directly involved LPAR's, but also of the LPAR's which are not directly involved.

Based on the factorial analyses a new prediction graph is created and shown in graph 33.



Graph 33: Graph of the measured response times (ELAT) versus the predicted response time (PRED) over one month in January 2009. Modeling was done with the dbms\_predictive\_analytics.predict routine, over also the indirect involved LPAR's.

The graph shows that the new created model is very well predicting the measured response time. When we start to interpret this data we come to the conclusion that there is a very strong indication that competition between the virtual machines, trying to get the CPU resources out of the free pool of "physical CPU" is the bottleneck. The virtual machine with the highest importance on the "physical CPU" is probably having a competition with the needed CPU's out of the free pool of "physical CPU".

Further investigation showed that the not directly involved LPAR's were running applications, which caused a competition on getting the free "physical CPU" out of the pool. Simply changing the priority and some changes in the run schedule of these applications solved the problem.

#### 13 Conclusions

After working with "Method-GAPP" for several years the method has proven to be a very powerful method to find out where in a technical architecture performance bottlenecks can be found, what will happen in future scenario's and what components outside the "chain" are influencing the enduser response time. The big advantage of the method is the fact that it doesn't rely on a specific platform, it can basically be used in any IT environment, so it can be said that the method is a real **General Approach** of **Performance Profiling**.

When we start at first with the method it looks very complicated but after getting some experience it is easy to use, especially in complex environments. A very good way to position the method is in an enterprise performance management situation. When in this position, data is collected enterprise wide and all collected data is put in one big performance metric data warehouse. Together with response time data of several important business processes over the entire enterprise, we can use the method at its full capacity. It is able to find relations between applications, which probably were not known and in this way the method can also be used to see how processes should be scheduled enterprise wide. Also strange performance degradations can be explained and tunings effort can be done on the performance bottlenecks found.

The method is also very successful in situations where limited data is available. In principle "Method-GAPP" can already do a good analysis only on the available data. The only thing what is most of the time not standard available, but necessary, is the response time data of the end-user process (business process) to be investigated. Although also this data is in some companies collected to see if the performance / availability SLA's are met. The used data mining can be used of Oracle Data Mining (ODM) but also from another data mining provider (e.g. Project-R, Mahout (Hadoop), etc.).

The data modeling makes a breakdown possible of the different involved components in the enduser response time. Having the formulas via the linear regression data mining can be of great help to see exact the impact of each individual component.

As we have seen in this whitepaper, the predictions of the models created via the method can help us to checkout if future investments would be worth the investment. This makes the method to be a powerful instrument of decision making for hardware / software investments or configuration changes.

To go ahead with the method fast it is very easy to start using tools like Oracle Enterprise Manager to use as a data source for the method. This decreases much the time of data gathering and data synchronization. In most of the situations these two steps take the most time.

As we further have seen is the usage of secondary component data like SQL statement response time or events data can be very powerful to checkout fast which SQL and/or which events are involved in specific end-user transactions. The power of this feature makes it possible to easily follow the end-user transactions over different tiers, databases, etc.

### 14 References

Optimizing Oracle Performance – Cary *Millsap 2003* 

Analyzing Computer System Performance with Perl::PDQ - Dr. Neil J. *Gunther 2005* 

**1** page 81, formula 2.66

**2** page 81, formula 2.64

Oracle® Data Mining Concepts11g Release 2 (11.2) E16808-06 – *Oracle 2011*